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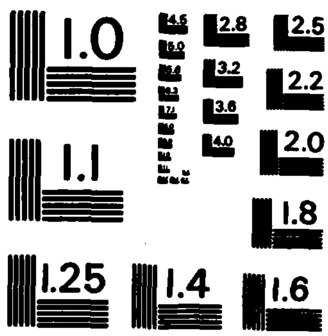

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A FIXED-CHARGE MULTICOMMODITY NETWORK FLOW
ALGORITHM AND A WAREHOUSE LOCATION APPLICATION

C. Harold Aikens
Richard E. Rosenthal

August 1985

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~~We~~ formulates a fixed-charge, multicommodity, minimum-cost network flow model, and fits the model to the distribution system design problem of a major Australian dairy producer. Due to its sparse demography and high standard of living, Australia is a particularly interesting place to apply distribution research. ~~We develop~~ an implicit enumeration algorithm which is capable of solving a large-scale problem and which indicates significant savings opportunities for the Australian firm. Additional keywords: optimization; integer programming; heuristic methods.

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Contents

A1

A FIXED-CHARGE MULTICOMMODITY NETWORK FLOW ALGORITHM AND A WAREHOUSE LOCATION APPLICATION

C. Harold Aikens and Richard E. Rosenthal
June 1985

We formulate a fixed-charge, multicommodity, minimum-cost network flow model, and fit the model to the distribution system design problem of a major Australian dairy producer. Due to its sparse demography and high standard of living, Australia is a particularly interesting place to apply distribution research. We develop an implicit enumeration algorithm which is capable of solving a large-scale problem and which indicates significant savings opportunities for the Australian firm.

This paper reports on our experience in developing a model, an algorithm and a computer program for the optimal design (and use) of a physical distribution system. The context of our work was the determination of warehouse locations for a major food products firm in Australia, but the (fixed-charge multicommodity network flow) model we describe is certainly not limited to decision problems of this type.

Questions of plant and warehouse location have long been studied by management scientists. (See Aikens [1984] for an extensive review or Table 1 for a brief selection of references.) The extent of practical implementation of management science/operations research in facility location is increasing but is by no means universal. Many firms have configured their distribution systems by evolution rather than by design. That is, incremental changes to their systems evolved in response to particular changes in demography, technology, acquisitions, divestitures, etc. Powers [1985] reports an interesting case where the accumulation of these changes over a 50-year period led to an extremely inefficient system,

Table 1. LITERATURE SUMMARY[1]

FEATURES[2]

	Capacities	Multi-echelon	Multi-comm.	Non-linear	Dynamic	Side Const.	Optimal
Kuehn & Hamburger [1963]	-	-	-	-	-	-	-
Efroymson & Ray [1966]	-	-	-	-	-	-	x
Spielberg [1969]	-	-	-	-	-	-	x
Khumawala [1972]	-	-	-	-	-	-	x
Erlenkotter [1978]	-	-	-	-	-	-	x
Cabot and Egrenguc [1984]	-	-	-	-	-	-	x
Gizelis & Samoulidis [1980]	-	-	-	-	-	-	x
Khumawala & Whybark [1976]	-	-	-	-	x	-	x
Kaufman, Eede, & Hansen [1977]	-	-	-	-	-	-	x
Tcha and Lee [1984]	-	x	-	-	-	-	x
Warszawski [1973]	-	-	x	-	x	-	-
Karkazis & Boffey [1981]	-	-	x	-	-	-	x
Khumawala & Neebe [1978]	-	-	x	-	-	-	-
Neebe & Khumawala [1981]	-	-	x	-	-	-	x
Van Roy & Erlenkotter [1982]	-	-	-	-	x	-	x
Akinc & Khumawala [1977]	x	-	-	-	-	-	x
Dearing & Newruck [1979]	x	-	-	-	-	-	x
Nauss [1978]	x	-	-	-	-	-	x
Geoffrion & McBride [1978]	x	-	-	-	-	x	x
Balachandran & Jain [1976]	x	-	-	-	x	-	x
LeBlanc [1977]	x	-	-	-	x	-	-
Geoffrion & Graves [1974]	x	-	x	-	-	x	x
Geoffrion, Graves, & Lee [1978]	x	-	x	-	-	x	x
Aikens & Rosenthal [1985]	x	x	x	-	-	-	x

[1] Includes integer programming-based models for warehouse location. (Discrete feasible set assumed.)

[2] Key to features (x = included):

Capacities -- allows upper and/or lower bounds on factory supplies, warehouse throughput and/or transportation flows.
Multi-echelon -- allows multiple warehouses between factory and customer.
Multicommodity -- allows for distinct modeling of multiple goods in shared distribution system.
Nonlinear -- allows for nonlinear transportation and throughput costs.
Dynamic -- allows for modeling of changes in customer demand and system configuration over multiple time periods.
Side constraints -- allows for additional constraints of general form, such as sole-sourcing.
Optimal -- supported by an optimizing algorithm. (Some but not all of the models have been field implemented.)

even though each step in the evolution made good business sense in its own time and place. Powers and several other authors (e.g., Geoffrion and Van Roy [1979]) argue convincingly that a comprehensive optimization-based analysis can lead to significant long-term savings far in excess of the cost of the analysis. (For a contrasting, simulation-based approach see Bowersox et al. [1972].)

We address the typical questions of such analyses in this paper:

(a) How many warehouses should be established?

(b) Where should the warehouses be located?

(c) What is the best routing of products from plants through warehouses and on to the customers?

Our most influential reference for this work was the optimization model reported by Geoffrion and Graves [1974] and extended by Geoffrion, Graves and Lee [1978]. A significant difference between the models reported by them and by us is that we allow more than one echelon of warehouses between plants and customers. We believe this extension is significant since it accommodates the common situation in which goods pass through a hierarchy of warehouses (e.g., from plant to district warehouse to regional warehouse to area warehouse). Our solution methodology also differs from Geoffrion et al., who use Benders' decomposition.

1. Background of Australian Case Study

The organization selected for the study is one of the leading manufacturers and distributors of ice cream products in Australia. In the early 1980s management interest in the configuration of the physical distribution network was particularly acute, due largely to the magnitude of

costs attributed to distribution-related functions (estimated to exceed $30 million annually) and to the following policy changes:

(a) A shift from conventional to highly automated warehousing.

(b) A merger with another Australian company which doubled the size of the national distribution network.

(c) The introduction of a new marketing strategy for small customers in metropolitan areas: telephone ordering replaced selling from the van.

1.1 Demographics

Australia is an especially interesting place to apply distribution research. A population approximately one-twentieth the size of the U.S. is spread over a land mass of similar size. Sixty percent of the people live in the seven capital cities (Sydney, Melbourne, Brisbane, Canberra, Adelaide, Hobart and Perth) which are all on or near the coast. Most of the remaining 40% live in other coastal areas, but a significant number of farmers, ranchers and miners live in the extremely sparse interior.

For several decades, Australians have enjoyed one of the world's highest standards of living. New products introduced in Europe and North America rapidly appear in Australian markets. The delivery of the goods (both domestic and imported) to sustain such a high standard of living to so sparsely populated a continent is very expensive. Hence, in comparison with most developed countries, Australia spends a large proportion of its GNP on distribution. (For 1974, the Productivity Promotion Council of Australia [1976] estimated this proportion at 15%.)

A study of the dairy industry provides an excellent example of why Australia is a particularly fruitful place to apply distribution research. Many parts of Australia are too arid for primary production. Even in wetter

parts, the small demand makes agrarian commercial ventures uneconomical. High distribution costs are inevitable, hence even small percentage improvements are very significant.

1.2 Product Line

The company under study produces over 100 distinct items, counting variations in flavor and package size. For the purposes of our model, these items were grouped into five commodities:

(a) Bulk ice cream and confectionaries.

(b) Take-home ice cream.

(c) House brand products.

(d) Loose pack stick/novelty items.

(e) Take-home stick/novelty items.

Customer demands are expressed in a variety of units, ranging from full pallets (known as "wraps" in the industry) for bulk purchasers to individual items for small accounts. Our model expresses all the demands in liters.

1.3 Distribution Network

The components of the distribution network are factories, warehouses, customers, and all of the permissable transportation links which join them. Figure 1 illustrates the node locations.

The corporate merger resulted in a total of seven ice cream factories on the network. For each of these factories, clearly defined minimum and maximum operating capacities were established for each of the five product groupings.

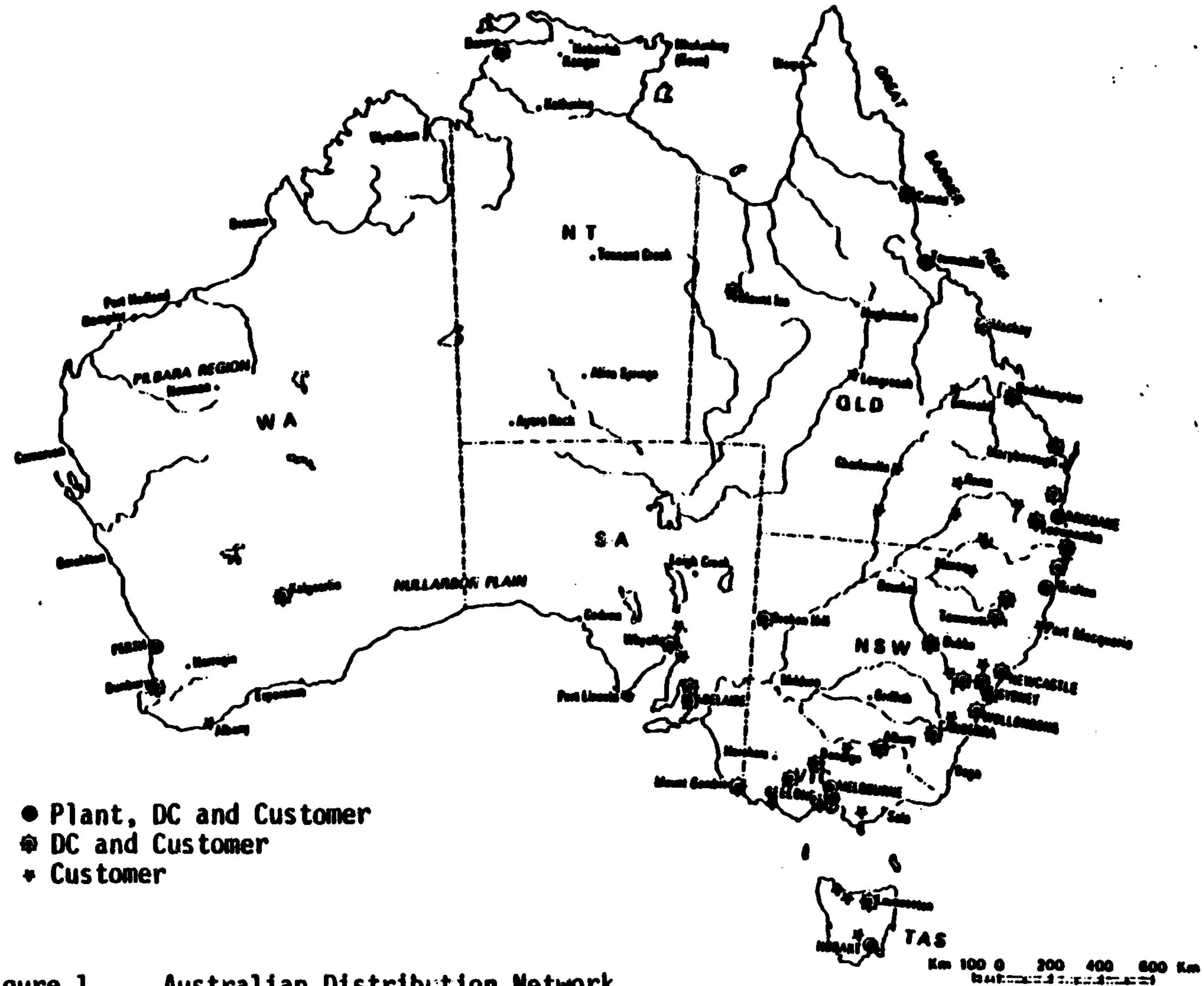

Figure 1. Australian Distribution Network.

A total of forty-three candidate warehouse sites, including existing sites, were selected. The warehouses were divided into two classes: major and minor. Major warehouses are defined as those permitted to receive replenishment stocks from any factory and any specified number of other warehouses. Minor warehouses are not permitted to receive supplies directly from a factory, they depend on other warehouses for supply. Each candidate warehouse has a maximum and minimum throughput level. Management is indifferent to which products contribute to the throughput in a particular warehouse, as long as the total amount of product fits within the given range.

Each customer belongs to one of seven market segments:

(a) Grocery chain warehouses which order in bulk.

(b) Contract warehouses which break up bulk orders for smaller retailers.

(c) Metropolitan small shops. Customers in this group are primarily sole proprietorships and include ice cream bars, delicatessens, corner shops--in essence, `mom and pop' stores. In the Australian economy, such stores are numerous; for example, in Brisbane (population 800,000) alone, it is estimated there are more than 2300 customers in this category.

(d) Caterers and food services within the areas served by major warehouses. Orders are filled on a preorder basis and deliveries are made by a fleet of small company owned trucks.

(e) Export to Papua New Guinea and Pacific Islands. Shipments are in container loads or smaller quantities by air or sea.

(f) Small orders. Customers in this grouping include small shops, schools, organizations, etc., that cannot be serviced by normal distribution channels (e.g., located in an isolated or remote area). Orders are packed in dry ice in special cartons and consigned to the customer by bus, rail, or truck.

(g) Staff sales. Employee stores are operated in certain locations.

For the purposes of this investigation, the export market, which represents a very small percentage of total sales, was ignored. The remaining markets are represented as 74 nodes on the network.

1.4 Costs

There are two types of costs in the analysis: variable charges for transport and warehouse throughput, and fixed charges for warehouse establishment and maintenance. Truck transport is the almost exclusive mode of shipment since door-to-door service minimizes the risk of product spoilage. Rail is used occasionally, but the savings in freight costs are not generally felt by management to justify the increased risks caused by delays and multiple handling. The transportation costs used in our analysis were based on over-the-road transport charter rates, with full loads, and in most cases, with trailers which are block stacked (that is, without pallets). Where customers require palletized shipments, the transportation costs reflect this.

Variable costs at warehouses include labor, inventory control, stock loss due to spoilage and pilferage, pallets, packing materials and some components of administrative costs. The fixed charges include interest, depreciation, salaries, utilities, engineering and maintenance. The company's amortization period was 10 to 30 years depending on the warehouse site.

For candidate warehouse sites which currently do not have warehouses, an additional amount is added to the fixed charges for construction. For existing warehouses an amount is subtracted from the

fixed charges to account for the costs that would be incurred in the event of closing it down.

In the next section, we present a model for minimizing the sum of all fixed and variable costs incurred subject to the satisfaction of customer demand and the observance of throughput limitations at the open warehouses. In the sections after that we present an algorithm for solving the model, and in the final section we report on the results of the algorithm for the Australian case problem.

2. Fixed-Charge Multicommodity Network Model

The general model which we adapted for the Australian distribution problem is the fixed-charge multicommodity capacitated transshipment (FC-MCTP) model, formulated as follows.

Indices:

$i \in I$, nodes

$j \in J$, directed arcs

$k \in K$, commodities.

Variables:

x_{jk} = flow of commodity k on arc j

$z_j = \begin{cases} 1 & \text{if arc } j \text{ has positive flow} \\ 0, & \text{otherwise.} \end{cases}$

Data:

c_{jk} = variable cost for flow x_{jk}

f_j = fixed-charge incurred if arc j has positive flow

b_{ik} = supply of commodity k at node i

ℓ_j, u_j = lower and upper capacities of arc j, if used.

FC-MCTP:

$$\min \sum_{jk} c_{jk} x_{jk} + \sum_j f_j z_j$$

subject to

$$\sum_{j \in F_i} x_{jk} - \sum_{j \in R_i} x_{jk} = b_{ik} \qquad \text{all } i,k \qquad \text{(flow balance)}$$

$$\ell_j z_j \leq \sum_k x_{jk} \leq u_j z_j \qquad \text{all } j \quad (\underline{\text{joint capacity}})$$

$$x_{jk} \geq 0 \qquad \text{all } j,k$$

$$z_j \in \{0,1\} \qquad \text{all } j$$

where F_i and R_i are the forward star and reverse star of node i. That means F_i is the set of arcs whose tail is i and R_i is the set of arcs whose head is i. Some notational remarks and assumptions:

(a) The index range for each summation and for each type of constraint is usually restricted in practice. For example, only 43 out of 1,612 arcs in the Australian case study have fixed charges. Consequently, only 43 binary variables (corresponding to warehouse open-or-close decisions) are explicitly defined. (All other z_j are implicitly set to 1.) Though not revealed in the notation above, the data structures of our implementation of the model take advantage of these and other efficiencies.

(b) For each commodity k, we assume that the total supply equals the total demand, i.e.,

$$\sum_i b_{ik} = 0 .$$

Otherwise, the flow balance equations would be inconsistent. (Any initial imbalance can be corrected in the standard way by adding a dummy node and slack arcs. This was done in our case problem.)

(c) The flows, variable costs, supplies and capacities are defined with respect to the same units of measure for each commodity (liters in our case). This is not a strict requirement. The alternative is to modify the joint capacity constraints with a commodity-specific weight applied to each x_{jk}. This would necessitate some minor changes in our algorithm.

The formulation of the Australian distribution problem as a FC-MCTP requires a standard modeling device (found, e.g., in Ford and Fulkerson [1962, p. 25]) for handling warehouse throughput. Any warehouse is represented by two nodes, say i and i+1, and a single arc $j = (i,i+1)$. The set of arcs which deliver goods to the warehouse are considered to ship to i, while the arcs which deliver goods from the warehouse are considered to ship from i+1. A binary variable on arc j then represents the open-or-close decision for the warehouse, and the capacities of this arc are the warehouse's throughput limits. Aside from this "node-splitting" device, defining the FC-MCTP model from the physical distribution network is totally straightforward.

A convenient, perhaps common, special property of the Australian distribution problem is that the variable flow costs on arcs are independent of commodity. Thus, we can replace c_{jk} by c_j in the model. This simplification has no significant algorithmic consequences, but it is helpful for computer implementation.

3. Algorithm

Our algorithm for solving the FC-MCTP is an implicit enumeration over the possible values of the binary vector z. In the facility location context, we refer to a proposed z as a configuration. Our case study has 43 potential warehouse sites. Hence, there are 2^{43} or about 8.8 trillion configurations. The determination of optimal flows for any one configuration is a formidable problem in its own right, namely, a multicommodity capacitated transshipment problem (MCTP). So, to repeat a familiar theme in integer programming, there would be no chance of ever solving the problem by exhaustive enumeration. Our experience with the implicit enumeration was most encouraging, however. An ε-optimal solution with $\varepsilon = 0.02$ was found by visiting only 2501 nodes in the enumeration tree and by completely solving only 30 of the MCTPs enumerated.

The generic structure of an implicit enumeration can be found in many standard references, such as Garfinkel and Nemhauser [1972]. The distinguishing features of our implementation are the methods employed for:

(a) obtaining an initial incumbent,

(b) obtaining an upper bound on the optimal flow cost for a given configuration z,

(c) obtaining a lower bound on the optimal flow cost for a given configuration z,

(d) obtaining lower bounds on partial solutions (fathoming by bounding),

(e) fathoming by infeasibility, and

(f) branching.

3.1 Heuristic for Obtaining an Initial Incumbent

We use a heuristic to obtain an initial incumbent solution. It is based on the idea of partitioning the distribution system into independent regions. In each region, the customer demands are aggregated and a set of warehouses with sufficient aggregate throughput capacity is opened. The warehouses are sorted according to their per-unit fixed plus variable cost when operating at full capacity, $(c_j + f_j/u_j)$. They are opened one at a time in this order until there is enough capacity for the region.

The heuristic is implemented with somewhat more sophistication than the description above implies. Details are omitted here but can be found in Aikens [1982, p. 126-132].

The idea of simplifying a problem by partitioning it into smaller parts is familiar not only to mathematical programmers but also to managers. The regionalization used in our execution of the heuristic for the Australian case study was based on existing managerial divisions. Without altering this regionalization, the heuristic found a new configuration that saved about $2 million, according to the model, over the existing configuration.

3.2 Upper Bounds on the MCTP

As noted earlier, each proposed configuration z defines a multicommodity capacitated transshipment problem, which we denote by MCTP(z). Its formulation is as given above for FC-MCTP, except that z_j is regarded as constant. (The obvious conditon $x_{jk} = 0$ if $z_j = 0$ is taken care of with the problem-generation data structures rather than an explicit joint capacity constraint.)

There are numerous algorithms available for MCTP(z). See Kennington and Helgason [1980, Chapter 4] for a review. Most of these methods are based on the observation that if the joint capacity constraints are ignored (or, more precisely, handled in some indirect way), then the resulting structure is a set of independent single-commodity flow problems. These problems are capacitated transshipment problems (CTPs), which are quickly solved by existing algorithms (e.g., Bradley, Brown and Graves [1977], Glover et al. [1974]).

One way of exploiting the observation is to allot to each commodity a portion of each arc's joint capacity and then solve for optimal flows within the allotments. This idea is called resource direction and is used, e.g., by Held, Wolfe and Crowder [1974] and Kennington and Shalaby [1977]. Formally, we chose an allotment $y = (\underline{y}_{jk}, \overline{y}_{jk})$ where, if $z_j = 1$, then

$$\sum_{k \in K} \underline{y}_{jk} = \ell_j$$

$$\sum_{k \in K} \overline{y}_{jk} = u_j$$

$$0 \leq \underline{y}_{jk} \leq \overline{y}_{jk},$$

or if $z_j = 0$, $\underline{y}_{jk} = \overline{y}_{jk} = 0$; and then we solve

$\underline{CTP_{UB}(z,y):}$

$$\min \sum_{j,k} c_{jk} x_{jk}$$

subject to flow balance and

$$\underline{y}_{jk} \le x_{jk} \le \overline{y}_{jk}, \qquad \text{all } j,k.$$

This problem is denoted $CTP_{UB}(z,y)$ for three reasons: its definition is affected by the choice of z and y, it is solvable as a set of independent CTPs, one for each commodoity, and it yields an upper bound on MCTP(z). We use the notation v[P] to mean the optimal value of problem P. The upper bound on MCTP(z) is

$$UB(z,y) = v[CTP_{UB}(z,y)] + \sum_j f_j z_j.$$

This is valid because $CTP_{UB}(z,y)$ is a restriction of MCTP(z). We obtain allotments y by the same procedure as Held, Wolfe and Crowder and Kennington and Shalaby. The least upper bound over all y considered is maintained as UB(z). This upper bound on MCTP(z) is of course also an upper bound on FC-MCTP; moreover, it can be used in conjunction with a lower bound to solve MCTP(z).

3.3 Lower Bounds on the MCTP

A second approach for exploiting the structure of MCTP(z) is to treat the joint capacity constraints in the objective function. This familiar idea is called Lagrangean relaxation (e.g., Fisher [1981] and Geoffrion [1974]). In this case it takes the form

$\underline{CTP_{LB}(z,\lambda)}$:

$$\min \sum_{j,k} (c_{jk} - \underline{\lambda}_j + \overline{\lambda}_j)x_{jk} + \sum_j (\ell_j\underline{\lambda}_j + u_j\overline{\lambda}_j)$$

subject to flow balance and

$$\ell_j \le x_{jk} \le u_j, \qquad \text{all } j,k \text{ s.t. } z_j = 1$$

$$x_{jk} = 0, \qquad \text{all } j,k \text{ s.t. } z_j = 0.$$

Here the Lagrange multipliers $\underline{\lambda}_j$, $\overline{\lambda}_j$ correspond to the lower and upper joint-capacity constraints on arc j. If $\lambda \ge 0$, then

$$LB(z,\lambda) = v[CTP_{LB}(z,\lambda)] + \sum_j f_j z_j$$

is a lower bound on MCTP(z). To prove this, let x^* be optimal in MCTP(z) and let v^* be the value of the $CTP_{LB}(z,\lambda)$ objective function at x^*. Then,

$$LB(z,\lambda) \le v^* + \sum_j f_j z_j \le v[MCTP(z)],$$

where the first inequality follows from the feasibility of x^* in $CTP_{LB}(z,\lambda)$ and the second inequality follows from $\lambda \geq 0$ and the feasibility of x^* in MCTP(z).

The greatest lower bound $LB(z,\lambda)$ over all λ considered is maintained as LB(z). We use two methods for obtaining trial values of λ, depending on whether we more recently solved a CTP_{UB} or a CTP_{LB}. In the first case, λ is imputed from the optimal duals in the CTP_{UB}. In the second case, we use the subgradient method in the same manner as Mulvey and Crowder [1979].

The combined use of UB(z) and LB(z) provides a means for solving MCTP(z). Another important use of LB(z) is in fathoming. If $LB(z) \geq UB$, where UB is the value of the incumbent solution to the FC-MCTP, then z can be discarded as a potential configuration even if we do not know the solution to MCTP(z). This is helpful, but of course our greater desire would be to avoid generating the inferior z altogether. The next two sections address this concern.

3.4 Lower Bounds on Partial Solutions (Fathoming by Objective Function Value)

Most of the time during an implicit enumeration, the binary vector z is only partially specified. That is, some z_j are fixed to 0 or 1 while other components are free. Given such a z, we define the problem $CTP_{LBC}(z,\lambda)$ to be the same form of relaxation as $CTP_{LB}(z,\lambda)$ with all free $z_j = 1$. Note that, if $\lambda \geq 0$, then

$$LBC(z,\lambda) = v[CTP_{LBC}(z,\lambda)] + \sum_{j \text{ fixed}} f_j z_j + \min_{j \text{ free}} f_j$$

is a lower bound on all the completions of z which allow at least one free $z_j = 1$. If $LBC(z,\lambda) \geq UB$, we can ignore all these completions. (Some refinements of this lower bound, taking capacities into account, are given in Aikens [1982, p. 107-108].)

3.5 Fathoming by Infeasibility

Another way of avoiding explicit consideration of configurations is fathoming by infeasibility, i.e., determining that a partial solution z has no feasible completions. The goal is to detect this condition before investing any effort in trying to solve an MCTP. We use four tests for this. They all involve comparing sums of capacities with sums of demands, a very inexpensive task. In the Austalian case study, these tests were extremely effective.

Denote the set of nodes representing customers in the distribution network by C, and let d_i be the total demand at $i \in C$, i.e.,

$$d_i = - \sum_{k \in K} b_{ik} .$$

The first two tests that follow assume that z is fully specified, the other three tests allow for free variables. The tests are:

(a) Reverse-Star-Configuration-Capacity Test: If

$$\sum_{j \in R_i} u_j z_j < d_i ,$$

then there is insufficient capacity to serve customer i, so z is infeasible.

(b) Aggregate-Configuration-Capacity Test: If

$$\sum_{i \in C} \sum_{j \in R_i} u_j z_j < \sum_{i \in C} d_i ,$$

then there is insufficient aggregate capacity to serve all customers, so z is infeasible.

(c) Reverse-Star-Completion-Capacity Test: If

$$\sum_{\substack{j \in R_i \\ j \text{ fixed}}} u_j z_j + \sum_{\substack{j \in R_i \\ j \text{ free}}} u_j < d_i ,$$

then z and all its completions are infeasible.

(d) Aggregate-Completion-Capacity Test: If

$$\sum_{i \in C} \Big(\sum_{\substack{j \in R_i \\ j \text{ fixed}}} u_j z_j + \sum_{\substack{j \in R_i \\ j \text{ free}}} u_j \Big) < \sum_{i \in C} d_i ,$$

then z and all its completions are infesible.

In the Australian case problem, over 90% of the nodes we examined in the enumeration tree were successfully screened out by this inexpensive battery of tests. As a result of these tests and the bounds of the previous sections, we only visited a minute proportion of the tree and we solved only a few MCTPs to completion.

3.6 Branching Rules

It is often remarked in the integer programming literature (e.g., Garfinkel [1979]) that the branching rule is the most crucial choice in the design of an implicit enumeration. In our case, we always fix $z_j = 1$ before fixing $z_j = 0$, so the question is which free warehouse should we open next?

We experimented with a total of eight branching rules and several ways of prioritizing them. We settled on the procedure described below. Let z be the partial solution from which we are about to branch.

(a) Reverse-Star-Capacity Rule. This rule gives first priority to any free warehouse which helps correct an infeasibility that was detected by the Reverse-Star-Configuration-Capacity test. If z, with all free $z_j = 0$, fails this test at node $i \in C$, then we branch on a free arc j whose head is in R_i. If no j or many j meet this condition, we consider the other rules.

(b) Maximum-Joint-Capacity-Violation Rule. In the second priority rule, we examine the solution to the relaxation $CTP_{LBC}(z,\lambda)$ (using the λ which yields the greatest lower bound on completions of z), and choose a free arc with the greatest violation of upper joint capacity.

(c) Maximum-Throughput Rule. If no free arc violates upper capacity in $CTP_{LBC}(z,\lambda)$, then we choose a free arc with maximum total flow. (This corresponds to maximum warehouse throughput in our application.)

Some of the additional branching rules that we tried are rules based on the regionalization concept employed in the starting heuristic or on a "greatest marginal savings" idea inspired by Akinc and Khumawala's [1977] Largest-Ω rule. (See Aikens [1982; p. 109-117] for details.) We did not find that the added work beyond the three simple rules above paid off. Perhaps more research will challenge this finding.

4. Solution to Australian Case Problem

The implicit enumeration algorithm whose components are described above has been programmed in FORTRAN and run on a DEC 10 computer. Our program is called MEDOS for "multiple echelon distribution optimization system." It uses GNET by Bradley, Brown and Graves [1977] as a subroutine for solving the capacitated transshipment problems CTP_{UB}, CTP_{LB} and CTP_{LBC}. Most of the time, the CTPs are started from an advanced basis.

The data for the Australian case problem has the dimensions:

5 commodities
7 plants
43 warehouses
74 customers

which results in a FC-MCTP model with

167 nodes
1612 arcs
7260 continuous variables
43 binary variables
835 flow balance equations
43 joint capacity constraints

Table 2 reports the solutions obtained by the complete algorithm and the starting heuristic on this problem. The complete algorithm saves approximately $13.5 million, according to our model, over the existing

TABLE 2. RESULTS FOR AUSTRALIAN CASE PROBLEM

	SOLUTION	
	Complete Algorithm	Starting Heuristic
Total Cost	$16,579,249	$27,930,467
Optimality Tolerance:		
Setting	0.02	
Value Achieved	0.017	
DEC-10 CPU Time:	96 minutes	35 seconds
Warehouse Closures:	Major: Toowoomba Newcastle East Sydney Canberra Geelong North Melbourne Ballarat Adelaide Perth Hobart	Major: Toowoomba Bundaberg East Sydney Canberra
	Minor: Darwin Launceston	Minor: Darwin Grafton
Warehouse Openings:	Major: Nambour North Sydney Woolongong Albury	Major: Nambour Wollongong North Sydney Albury East Melbourne
	Minor: Tamworth Bathurst Elizabeth	Minor: Lismore Tamworth Bathurst Whyalla Elizabeth Kalgoorlie Bunbury

TABLE 3. COMPUTATIONAL STATISTICS FOR AUSTRALIAN CASE STUDY

	SOLUTION	
	.0025-Optimal	.017-Optimal
Enumeration-Tree Nodes Visited		
Number	13,108	2501
% of Maximum	1.5×10^{-7}	2.9×10^{-8}
CTPs Solved	83,850	26,990
Number of Successful Screenings by:		
Reverse Star Configuration Capacity Test	11,535	2,457
Aggregate Configuration Capacity Test	198	0
Reverse Star Completion Capacity Test	85	38
Aggregate Completion Capacity Test	9	0
Configuration Lower Bound	1,344	14
Completion Lower Bound	6,459	1,205

distribution system, whose total costs were estimated at $30 million.

The objective function value obtained by the starting heuristic is $11 million worse than the value obtained by the complete algorithm. This is a convincing illustration of Geoffrion and Van Roy's [1979] warning about the danger of relying upon heuristics for corporate planning.

The optimality tolerance referred to in Table 2 is the value of (UB-LB)/LB, where LB and UB are the greatest lower and least upper bounds on v[FC-MCTP]. The maximum allowed value of this ratio is an input parameter in our program; it is reported in Table 2 along with the value achieved. A higher tolerance setting generally leads to a shorter running time. As an experiment, we ran the algorithm with the very low tolerance setting of 0.0025 and achieved this value after 13 hours on the DEC-10. The objective function improved by another $270,000. This amount would obviously offset the additional computing cost, if it were realized, but a planning model in practice is usually run very many times before any action is taken. Most of these runs are easier to solve than the original problem, because they have a large proportion of the binary variables pre-assigned to fixed values. Nevertheless, we would not consider our experimental run with all z_j free and with $\varepsilon = 0.0025$ to be practicable.

Table 3 reports some computational statistics which indicate the relative effectiveness of various aspects of our complete algorithm on the Australian case problem. The most important overall conclusion from this table is that, even with very strict optimality tolerances, our algorithm is very successful at avoiding explicit enumeration of undesireable configurations.

It is of course very difficult to compare the performance of algorithms except under carefully controlled conditions. Lacking these conditions, we can only make some parallel observations without making conclusions. Ali, Helgason and Kennington [1982] present an FC-MCTP model and an algorithm for designing a military logistics system. The logistics network has 60 nodes, 3540 arcs, and 12 commodities. The most important feature to compare is the number of fixed-charge arcs, which determines the number of binary variables. The logistics model has 25 of these, so the number of configurations to be considered is 2^{25} (about 34 million), compared with 2^{43} (about 8.8 trillion) in the Australian model. Ali et al. report spending 23 CPU hours to solve the problem on a Cyber 73, a computer which for scientific computing is approximately 7 times faster than the DEC-10.

The software we have developed includes features for convenient data modification and reoptimization. These are essential for putting any algorithmic and modeling research to practical use in a managerial setting. Considering the large number of changes to the existing configuration which were recommended by the model, we would advocate many more model runs before implementing any changes. It seems particularly important, given our results, to go back and question whether the fixed charge components for warehouse openings and closings were sufficiently high. The closing costs are particularly important to analyze parametrically, since they must incorporate, albeit subjectively, some loss of goodwill and some cost for the disruption of employees' lives.

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